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On an inequality of Chern numbers coming from anomaly calculations associated to ABJM(Aharony-Bergman-Jafferis-Maldacena) models

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1 Introduction

Whether there exists "expected" Miyaoka-Yau type inequlity,especially including $c_i(X)(i > 2)$, on higher dimensional algebraic variety X of general type,have been posed by Y.Kawamata and N.Nakayama,since the proofs of flip theorem and the existence of the minimal models for 3-folds were established by S.Mori, because of the abundance conjecture on higher dimensional case.Recently,A.Bayer et al [3] showed the similar inequality as Bogomolov-Gieseker type one on projective 3-folds,based on the calculations of black-hole entropy on the context of the recent developments of AdS/CFT (or,gauge/gravity) correspondences.Considering these correspndences,M-theory are expected to have maximal supersymmetry as gravity theory in the area of low-energy.The ABJM (Aharony-Bergman-Jafferis-Maldacena) model [2] in the title,which is expected to be one of such theories having these properties,is a relativisitic model with 2-dimensional space and time as a M2 brane in 11-dimensional model having 8-dimensional freeness, which gives explicit values of free enegies on gauge theories by scattering amplitudes from the side of bulk (gravity theory) based on AdS/CFT correspondences.

2 Nambu brackets as descriptions of interactions of M2 branes

Considering Lagrangians for M2 branes as dynamics or statistics of them,the Chern-Simons theory as its action $S_{CS} = \dfrac{k}{4\pi}\displaystyle\int \mathrm{Tr}(A \wedge dA + A^3)$ under a 'tHooft limit of large N with a fixed ratio N/k.where k an integer after quantization,where A is gauge field as 1-form,is required to give description of interactions of M2 branes without field strenghts,because the induced equation of motion $F_{\mu\nu} = 0$.However, when the covariant derivatives for scalar fields X^I are introduced as $D_\mu X^I = \partial_\mu X^I + iA_{\mu ab}[T^a, T^b, X^I]$,based on the gauge invariances, where $[T^a, T^b, T^c] = if^{abc}_d T^d$ is called Nambu bracket [7] (this bracket is similar to the one in [8]),the Lie 3-algebra induced by such Nambu brackets is difficult to describe $N > 2$ M2 branes but for 2 M2 branes,under the gauge invariances [4, 5].So,O.Aharony et al [2] posed the classical moduli theory with $\mathrm{U}(N) \times \mathrm{U}(N)$ theories under the smaller supersymmetry,and with not receiving quantum corrections as Chern-Simons terms $S_{CS} = \dfrac{k}{4\pi}\displaystyle\int (A_{(1)} \wedge dA_{(1)} - A_{(2)} \wedge A_{(2)})$ for gauge transformations $A_{(i)} \rightarrow A_{(1)} - d\Lambda_{(i)}$ $(i = 1, 2)$ and $C_I \rightarrow e^{i(\Lambda_{(1)}-\Lambda_{(2)})}C_I$ for $A_{(i)}$ field variables and C_I superfields (two of them are chiral and the others are anti-chiral). Then,taking boundary contributions to Chern-Simons terms on $\mathbb{C}^4$, $\delta S_{CS} = \dfrac{k}{2\pi}\displaystyle\int_{boundary} (\Lambda_{(1)} \wedge F_{(1)} - \Lambda_{(2)} \wedge F_{(2)})$ by Stokes theorem, where $F_{(i)}(i = 1, 2)$ are field strenghts as 2-forms,the gauge field strenghts are quantized as $\displaystyle\int F_{(i)} \in 2\pi\mathbb{Z}$ on any closed 2-manifold.So, $\Lambda_{(i)} = 2\pi n/k$ for some integer n for the action to transform by 2π,then the moduli space is $\mathbb{C}^4/\mathbb{Z}_k$, where $\mathbb{Z}_k$-symmetry acts as $C_I \rightarrow e^{2\pi i/k}C_I$.

3 Axial anomaly for Chern-Simons terms in the deformation of NS5-branes and D3-branes

Main Results We consider axial anomaly associated to the brane construction via type IIB strings for Chern-Simons terms with the moduli $\mathbb{C}^4/\mathbb{Z}_k$, associated to probing $(1, k)$ D5-branes on the brane configurations of 2 NS5-branes and N D3-branes with supersymmetry fixing $(1, k)$ D5-branes in the 59-planes in (0123456789)-space [2] as non-relativistic case, motivated by [2, pp.17] with regards to the applications to elliptic fibered Calabi-Yau 4-folds $(\mathbb{T}^2)^4$ with the moduli $\mathbb{C}^4/\mathbb{Z}_k$ as geometric assumptions to have an inequality of Chern classes induced from the anomaly polynomials.So,for (p,q) D5-branes,the Chern-Simons action is $\dfrac{a}{g_4^2}\dfrac{p}{q}\displaystyle\int d^3x\epsilon_{\mu\nu\lambda}A^\mu\partial^\nu A^\lambda$ [6] where a is constant determined to be $g_4^2/4\pi$,then the anomaly polynomial [1] $I_{10}(F) = \dfrac{i^5}{(2\pi)^4 5!}\mathrm{Tr}F^5$ where $F := \epsilon_{\mu\nu\lambda}A^\mu\partial^\nu A^\lambda$ is the field strenght with the signature $\epsilon_{\mu\nu\lambda}$. For covariant derivatives $D_\mu X^I = \partial_\mu X^I + iA_{\mu ab}[T^a, T^b; X^I]$ with $[A, B; C] := AC^\dagger B - BC^\dagger A$ [2] weaker than [4, 5], because that the axial anomaly is given by $\partial_k \mathrm{Tr}(\gamma^k\gamma^5\mathbb{D})$,where γ^k, γ^5 Dirac gamma matrices, for Hermitian operator $\mathbb{D} := \gamma^\mu D_\mu = \gamma^\mu(\partial_\mu - \sum_a(iV^a_\mu + iA^a_\mu T^\mu\gamma_5)) =: \gamma^\mu(\partial_\nu - iV_\nu - iA_\nu\gamma_5)$ where V_ν vector fields and A^a_ν axial-vector fields, then the resulting anomaly polynomial is $\tilde{I}_{10}(\tilde{F}) = \mathrm{Tr}\{(\epsilon_{\mu\nu\lambda}\gamma^k\gamma^5\mathbb{D}A^\mu\partial^\nu\gamma^k\gamma^5\mathbb{D}A^\lambda)^5\}$ up to constant multiplication for the axial gauge tranformed $\tilde{F}$.

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